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EFFECTS OF STOCKING ON SITE MEASUREMENT AND YIELD OF SECOND-GROWTH PONDEROSA PINE IN THE INLAND EMPIRE

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OGDEN, UTAH
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(A Dissertation submitted in partial fulfillment of the requirements for the Degree of Doctor of Forestry in the School of Forestry of Duke University.)

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INTRODUCTION

To a forester, the term site embodies an appraisal of all the environmental factors of an area that combine to determine its productive capacity for a specific forest species. Since the term reflects the influence of many variables, the more important of which are soil, moisture, temperature, and length of growing season, the measurement of site has proved to be complex and often inaccurate.

Site has been evaluated, for example, on the basis of the soil profile alone by correlating certain soil properties with the measured success of a tree species growing thereon. To the degree that soil characteristics reflect the combined influence of the many factors of the environment, they are valuable in delimiting divisions of productivity. The success of the edaphic approach has varied with locations and species.

Another method of site evaluation has been the use of plant indicators--ground species that have been found consistently associated with a particular quality of forest growth. A thorough knowledge of the ecology of plant associations is, of course, essential to the success of this method. Because of the variability and complexity of plant associations, only broad divisions of site quality are usually expressed, using such descriptions as "good," "medium," or "poor." Both the soil and the plant indicator methods of forest site appraisal are especially useful on areas that do not at the moment support forest growth or which support uneven-aged or disturbed stands.

On areas where even-aged, undisturbed stands of timber are growing, the direct measurement of forest growth has been by far the most used and most dependable index to site quality. The one measure of growth found to be most independent of stand factors and consequently most reliable for site evaluation has been height of the dominant stand in relation to its age. The relationship of height and age has been expressed in the familiar term "site index," which refers to the height in feet of the dominant stand at a chosen index age--commonly 50 years, but in ponderosa pine, 100 years. Thus the well-known family of site curves has evolved which shows dominant heights over a range of age for each of several site indices.

Foresters realize, however, that site index is not a panacea for site evaluation under all conditions. They read site index to the nearest foot, but they know that it is not that accurate; an unknown error must be accepted because of the many undetermined variations such as stand density, genetical strain of the species, insect and disease depredation, and ground fires. One of the most important of these variables and one that can be measured is stand density.

Dominant stand height has commonly been considered rather independent of stand density and therefore reliable as a measure of site quality. Forestry literature abounds with references to this independence of height to stocking for many species and conditions. Yet, many other examples have been reported where stand density has either stimulated or retarded height growth. Among those species that have exhibited this phenomenon is ponderosa pine (*Pinus ponderosa* Laws.). Observations of ponderosa pine in the Pacific Northwest, by the writer and by others, have confirmed the fact that dense stocking materially reduces height growth on poor sites and that this reduction is sufficient to impair the accuracy of site quality measurements.

The first objective of the present study is to show the magnitude of the effects of stocking on the height of ponderosa pine and to develop adjusted site index curves for use in stands having various degrees of stocking. The proper evaluation of site is basic to the prediction of forest yields--one of the foremost problems in forest management.

The second objective of this study is to present a method of growth prediction for second-growth ponderosa pine which is especially applicable to understocked stands.

HEIGHT AND STAND DENSITY--A REVIEW OF LITERATURE

The commonly accepted fact that dominant stand height is quite independent of stand density is supported by several well-designed spacing studies for several forest species. Bramble *et al*. (1949) reported a study of planted red pine (*Pinus resinosa* Ait.) in Pennsylvania on a good site. Planted at four spacings ranging from 5 x 5 feet to 10 x 10 feet, the dominant stands were the same height after 25 years. No indications of stagnation occurred regardless of spacing. Ware and Stahelin (1948) made similar observations on plantations of three southern pines, slash pine (*Pinus elliottii* Engelm.), loblolly pine (*Pinus taeda* L.), and longleaf pine (*Pinus palustris* Mill.) in Alabama where seven spacing treatments ranged from 4 x 4 feet to 16 x 16 feet. Although considerable differences in volume growth and products were apparent in stands having different spacings after 14 years, height of dominant trees was not affected. Here again, the site index was high and tree dominance well expressed throughout.

Slash pine was experimentally planted in Louisiana at the rate of 250, 1,150, 1,600, and 2,500 trees to the acre, and after 14 years there was no difference in dominant heights due to stocking (Mann and Whitaker 1952). In a 9-year-old plantation of jack pine (*Pinus banksiana* Lamb.) in Michigan, Rudolf (1951) reported no differences in dominant height due to spacings that varied from 1½ x 1½ feet to 9 x 9 feet. The plantation was on a good site for jack pine, and 9 years was not sufficient time for even very close spacing to cause stagnation. Another plantation of jack pine in lower Michigan spaced from 4 x 4 feet to 10 x 10 feet had equal dominant heights after 25 years (Ralston 1953).

The significant facts in each of these plantations are that the sites were good and dominance was well expressed either by virtue of the good sites or because of inherent growth characteristics of the species.

That density of stocking does affect height growth of certain forest trees under certain conditions is also well documented. The effects, however, are not always in the same direction; some species react by being taller than average under condition of dense stocking whereas others are stunted. The paradox can be explained by the inherent growth habits of the species concerned and by the quality of the site on which they grow.

Gaiser and Merz (1951) found that in even-aged white oak (*Quercus alba* L.), over a wide range of site quality, dense stands contained taller trees than open stands. Given the space, the crown of this broadleaf species tends to extend outward rather than upward. Also, working in oak forests, Gevorkiantz and Scholz (1944) found height growth retarded in understocked stands. They developed a method of site evaluation based on volume rather than height.

In a red pine plantation in northern Lower Michigan, spacings were 4 x 4, 6 x 6, 6 x 8, and 8 x 9 feet. Ralston (1954) reports that after 35 years dominant heights ranged from 25.4 feet in the 4- x 4-foot spacing to 31.5 feet in the wide spacing. The soil was Grayling sand, one of the poorest red pine soils of the region. Another spacing study reported by Adams and Chapman (1942) involving jack pine, red pine, eastern white pine (*Pinus strobus* L.), pitch pine (*Pinus rigida* Mill.), and Scotch pine (*Pinus sylvestris* L.) showed that close spacing retarded height growth of red pine and eastern white pine, species more exacting in site requirements, to a greater degree than it did in jack, pitch, and Scotch pine, species better adapted to poorer sites.

Effects of density on height are also sometimes apparent in natural stands. Turner (1943) observed that in even-aged old field loblolly pine and shortleaf pine (*Pinus echinata* Mill.), dense clumps were taller than adjacent open clumps. Tree crowns were smaller in the dense clumps, and growth was concentrated in the terminal shoots. Old fields are generally very favorable sites for these southern pines. Clumps of red pine in Michigan were observed by Shirley and Zehngraff (1942) to be taller in the center where density was greatest. Soil conditions apparently were better in the denser stands because the ground was shaded.

Thinning studies have provided opportunities to measure height growth under various degrees of stocking. A very dense 5-year-old jack pine stand in Minnesota was thinned to various spacings. Roe and Stoeckeler (1950) report little effect on height growth due to spacing after 5 years, but they observed large differences in diameter growth and crown size. Jack pine heights seem consistently to be less affected by stand density than heights of red pine. Engle and Smith (1952) report that height growth of both an overstocked natural stand of red pine and a 40-year-old plantation with a 5- x 5-foot spacing was improved 10 years after a moderate thinning.

Turning to ponderosa pine, Weaver (1947) gives an account of a 40-year-old stagnated stand on a poor site in eastern Washington, a portion of which had been thinned by fire at the age of about 10 years. The unthinned portion of the stand had a height of 12.3 feet as compared to 32.2 feet for the thinned portion. In the Southwest, Krauch (1949) found that in thinning trials of ponderosa pine stands,

both the crop and the noncrop trees were on the average 2 feet taller in the thinned plots than in the unthinned plots for each 10 years following thinning. Moisture is the most important factor limiting growth in this locality.

In a natural stand of ponderosa pine in northern California, Baker (1953) observed wide variations in stocking due to distance from the seed source. On apparently uniform site, height growth was superior in the open portion of the stand. In summarizing ponderosa pine thinning studies in the Pacific Northwest, Mowat (1953) discusses height stunting in overcrowded stands and its effect on site evaluation.

Two general observations are possible from the several reports reviewed. First, the effect of density of stocking on height growth varies with the species concerned and is related to their basic growth habits and abilities to express dominance. Second, when height growth is retarded by density of stocking, it is most apparent on poor sites where root competition is high.

THE PONDEROSA PINE TYPE IN THE INLAND EMPIRE

Ponderosa pine is the most important forest species in the Inland Empire, an area rather indefinitely bounded by the Continental Divide in Montana, the desert of central Washington, the Salmon River in central Idaho, and the Canadian border. Second only to western white pine (*Pinus monticola* Dougl.) in lumber quality, ponderosa pine surpasses white pine in gross value of product because of its abundance and wide distribution. In the Inland Empire, ponderosa pine grows in nearly pure stands on the lower foothills and valley floors where precipitation ranges from 15 to 30 inches per year, in an altitudinal range of 1,500 to 3,500 feet (fig. 3). It thrives on coarse, well-drained soils, such as sandy alluvium, gravelly or sandy till, and loams having high stone content. On the poorer, drier sites, ponderosa pine stands have very little understory of brushy species and are famous for their parklike appearance. As sites improve, understory brush becomes more plentiful. Figure 1 shows a comparison of a 120-year-old stand with site index 83 having only grass and needles on the forest floor and a 90-year-old stand of site index 100 with a rather dense understory of brush and Douglas-fir reproduction. As moisture conditions improve, either because of higher altitudes or favorable exposures, pure ponderosa pine stands give way to Douglas-fir (*Pseudostuga menziesii* var. *glauca* (Beissn.) Franco) and other more mesophytic species. Often the ponderosa pine stands found on such sites are subclimaxes resulting from past fires; as the stands close and environmental conditions change, an understory of Douglas-fir appears. Other common associates of ponderosa pine are lodgepole pine (*Pinus contorta* Dougl.), western larch (*Larix occidentalis* Nutt.), and grand fir (*Abies grandis* (Dougl.) Lindl.).

Mature stands of ponderosa pine are typically uneven-aged. The species is long-lived and generally reproduces well in openings, so that most stands contain several age classes often present in small, even-aged clumps. The stocking, too, is irregular; frequent openings, sparse stands, and very dense clumps appear on every acre.

Early logging activity in the Inland Empire upset the natural stand development by subjecting large areas to clear-cutting and burning; as a result, many even-aged stands of second-growth pine have replaced the virgin stands. Some of these stands are in the 80- to 90-year age class and are reaching the lower limits of saw log merchantability. Others are suitable size for pulpwood, posts, and poles; they are being recognized as a valuable timber asset to this region. Although generally even-aged, these second-growth stands still reflect the irregular growth habit of the species by being extremely variable in stocking. Frequent ground fires have further accentuated the irregularity of stocking. Figure 2 shows a comparison of two stands, both approximately 55 years old, which have had different densities during their lives, which have resulted in differences in tree size and condition. Because moisture is nearly always the most important limiting factor to the growth of ponderosa pine in this region, site quality varies with every ridge, swale, and change of exposure, and thus produces still another form of irregularity.

FIELD MEASUREMENTS

SOURCE OF DATA

Data for this study were gathered from temporary sample plots located in even-aged stands of second-growth ponderosa pine throughout the Inland Empire. Figure 3 is a map of the region showing the ponderosa pine type and the sample plot locations. Each plot was carefully selected for its uniformity of site, age, and density, but in the aggregate the plots represent a range in ages from 25 to 125 years, site indices from 40 to 130, and stand densities from very open to dense thickets. Because the variables--age, site, and stocking--were to be evaluated in site and yield determinations, it was important that

"A"

"B"

Figure 1.--A shows a 90-year-old stand of ponderosa pine of site index 100 with an understory of brush and Douglas-fir reproduction typical of good sites. B shows another stand of the same species, age 120, site index 83, having only grass and needles on the forest floor--a condition characteristic of stands on medium and poor sites.

"A"

"B"

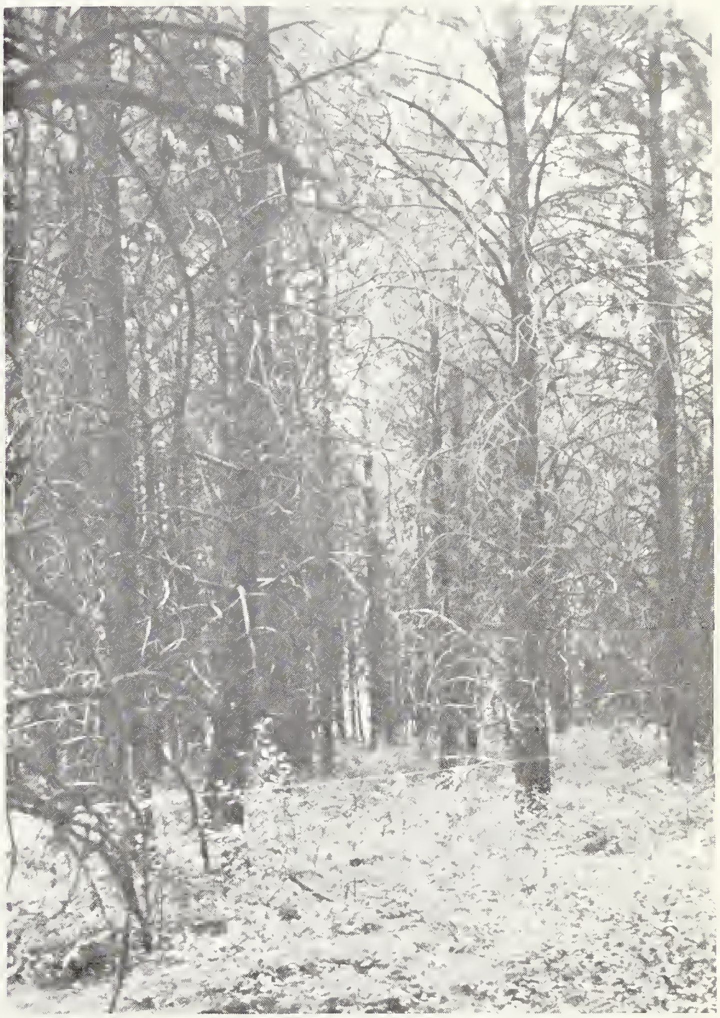

Figure 2.--These two stands of ponderosa pine, 55 years old, have had different densities during their lives, which have resulted in differences in tree size and condition. A shows a stand with 2,500 trees per acre whose stocking percentage is 115. B shows a stand with 400 trees per acre with a stocking percentage of 83. This stand occupies a better site than the one shown in A, as the surface vegetation indicates.

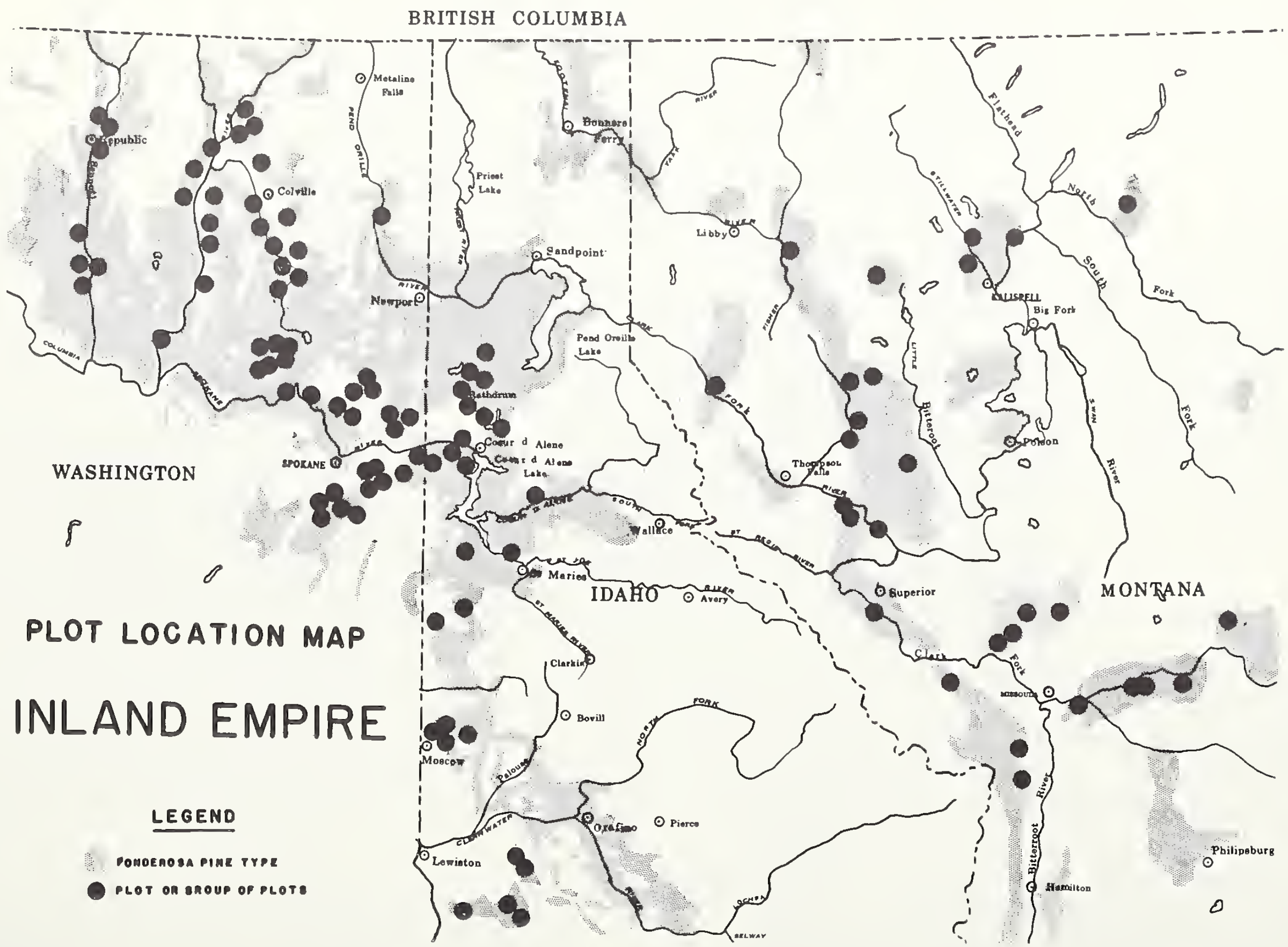

Figure 3.--Map of the Inland Empire showing the ponderosa pine type and the location of plots or groups of plots measured in this study.

each plot represent uniform conditions of each. A plot of average stocking, for example, could not be a combination of a dense clump and an opening, because density effects on growth would then be obscured. To find plots having uniform stocking was a difficult phase of the study.

Plot sizes ranged from one-fortieth of an acre to one acre; the majority were one-fifth acre in size. Attempt was made to have from 80 to 100 trees per plot; thus, tree size governed plot size. Small plots were confined to dense, young stands.

During the field seasons of 1953 and 1954, 72 sample plots were measured specifically for this study. In addition, data from 49 other plots measured by the writer in the same region in connection with another study were used (Lynch 1954). Finally, 86 Inland Empire plots were selected from among those measured for Meyer's interregional yield study for even-aged ponderosa pine (Meyer 1938). These 207 plots form the basis for the present study, each plot supplying the following data:

1. Age of dominant stand.
2. Curve of height over diameter based on 20 to 30 measured heights.
3. Stand tally of all trees 0.6-inch d.b.h. and larger by crown classes.
4. Miscellaneous information on soil and surface vegetation.

Site index for each plot was determined from Meyer's (1938) site curves, using dominant height read from plot height curves for average d.b.h. of all dominant and codominant trees. All plot data, such as basal area, volume, and number of trees, were converted to an acre basis.

PAIRED PLOTS

The irregularity of stocking in second-growth ponderosa pine stands gives an opportunity to observe striking differences in height on adjacent stands of the same age and apparently on the same site but with different densities. Very abrupt changes in stocking are found frequently and are probably caused by past ground fires as well as by irregular seeding. Significant differences in growth predictions can result from errors in site evaluation caused by such variations in stand densities. Differences in site index estimates as great as 20 units are often obtained from these adjacent stands which, judged on the basis of soil, slope, aspect, and surface vegetation are growing on identical sites. Referring to normal yield tables, one finds that a spread of only 10 site index units gives differences in growth predictions as high as 24 percent. Such variations, of course, have economic significance.

To evaluate the effect of stocking on height, hence on site index determination, requires an independent and true measure of the site quality. The two possible independent measures--soil profile and indicator plants--have not been explored sufficiently in this region to be used. A system of paired plots was therefore adopted in which two plots were selected side by side, identical in age and apparent site quality, but differing in stocking. One plot in each pair, designated as the plot, supplied all the usual measurements for the study, while the adjacent plot, referred to as the control, supplied an independent measure of site index. Great care was taken to select pairs that were believed to be on the same actual site. Each control was required to have average stocking.

It was difficult to find areas that fulfilled the requirements of paired plots. In two seasons' field work only 24 such pairs were found that were judged suitable. Figures 4 and 5 show stands representative of the sharp changes in stocking and height with otherwise identical characteristics. Only in a forest type in which root competition is a major critical factor would such marked differences occur.

The 24 plots, with their controls, form the basis for the site curve adjustments in this study. All 207 plots were used in the general site index equation.

ADJUSTING SITE INDEX BY STAND DENSITY

MEASURING DENSITY

Of the several measures of stand density that have been used, tree-area ratio, proposed by Chisman and Schumacher (1940), is especially applicable to a group of plot data. It consists of a quadratic equation, fit by the method of least squares, to plot data expressed on a unit area basis using such variables as sum-of-diameters, sum-of-diameters-squared (or plot basal area), height, age, or other variables that might affect stocking. The calculated equation expresses density of stocking as a proportion of the average density of stocking represented by the aggregate plot data.

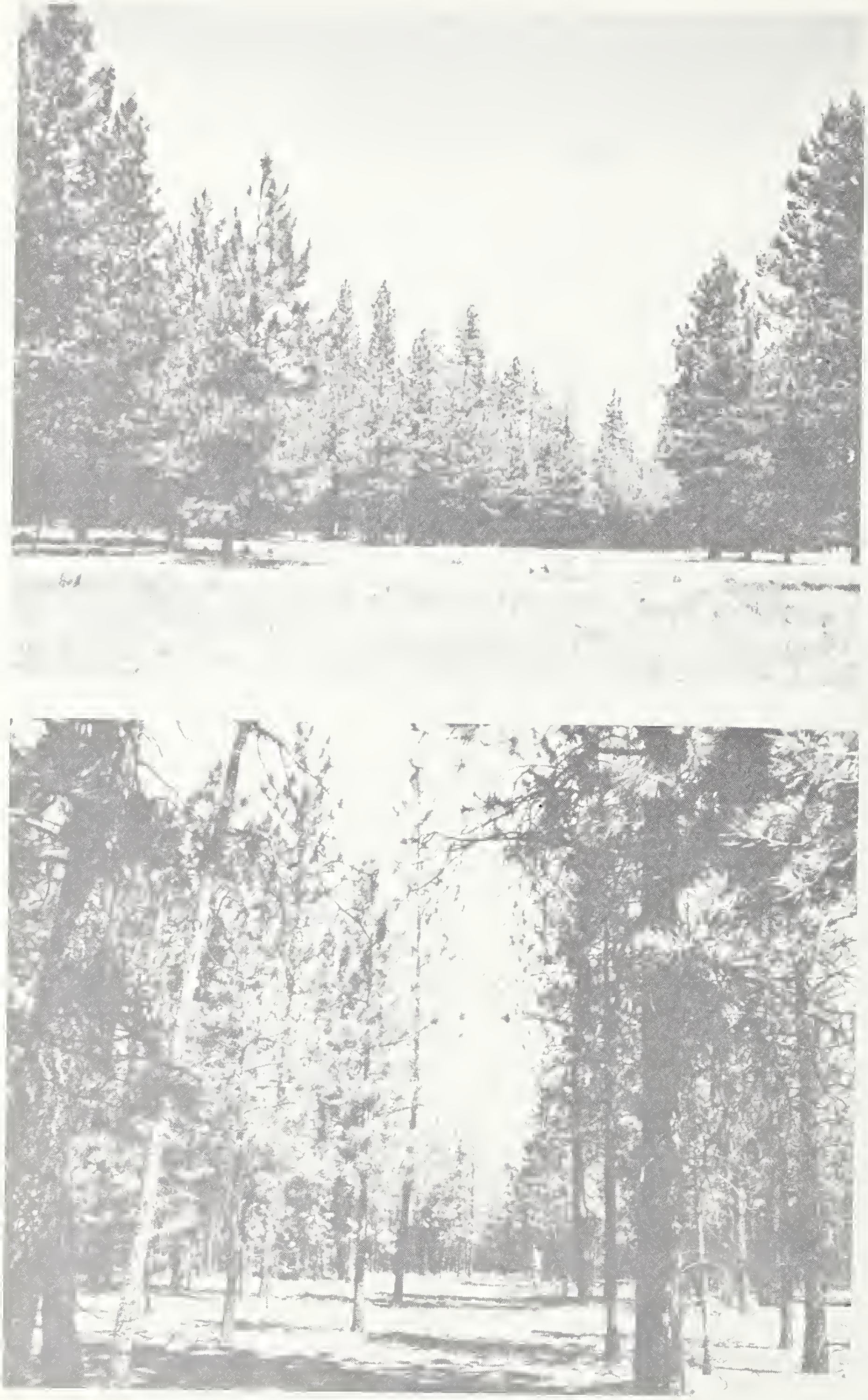

Figure 4.--Two views of even-aged ponderosa pine, about 40 years old, showing considerable differences in heights between open grown trees and trees in dense stands.

Figure 5.--This 25-year-old even-aged stand of ponderosa pine shows a striking difference in height due to stocking.

The tree-area ratio equations used by Chisman and Schumacher (1940) on loblolly pine plot data and the one used by Gaiser and Merz (1951) on white oak data contained these variables: sum-of-diameters, sum-of-squared-diameters, and number of trees. The expressions were found to be independent of site index and age for those particular forest types. Lexen (1939) used the tree-area ratio for a space-requirement study of ponderosa pine in the Southwest in which he reduced the equation to sum-of-squared-diameters only, that being the most significant variable.

It was suspected that stocking of ponderosa pine stands in the Inland Empire might vary with site and age; accordingly, the two variables, height and age, were included in the tree-area ratio expression. Basal area was used rather than sum-of-squared-diameters giving the equation,

$$S = b_1(N) + b_2(B) + b_3(BH) + b_4\left(\frac{BH}{A}\right) + b_5\left(\frac{B}{A}\right)$$

where S = stocking percentage
N = number of trees per acre
B = basal area per acre
A = age of dominant stand
H = height of dominant stand
b_1 to b_5 = coefficients to be computed.

A more detailed explanation of the stocking equation is given in Appendix I.

Number of trees per acre was found to be the least significant of any of the variables and was dropped from the equation. After calculating the regression coefficients, the equation became:

$$S = 0.2918(B) + 0.0065(BH) - 0.6467\left(\frac{BH}{A}\right) + 29.7520\left(\frac{B}{A}\right).$$

Factoring out B, the equation takes the form,

$$S = B\left[0.2918 + 0.0065(H) - 0.6467\left(\frac{H}{A}\right) + 29.7520\left(\frac{1}{A}\right)\right] \qquad (1)$$

in which the expression within the brackets is the stocking per square foot of basal area.

The stocking percentage calculated by equation (1) is the ground area that a stand of given age, height, and basal area would have utilized in an average-stocked stand relative to the actual ground area of the stand itself. A stocking percentage of 100, as used in this study, indicates a density equal to the average of all the data represented. It should not be confused with 100 percent of normal, because not all the stands chosen in this study were fully stocked as they are in a normal yield study.

Examples of the stocking percentages calculated by equation (1) from normal yield table data for second-growth ponderosa pine (Meyer 1938) appear in table 1. They are generally above 100, showing that the stocking of the plots measured for the normal yield study was higher than that of the plots in the present study, except in the older ages of the poorer sites.

Table 1.--Stocking percentages calculated from normal yield table data for second-growth ponderosa pine for four age and six site classes

Age	Stocking percentage when site index is: 50	60	70	80	90	100
			Percent			
40	109	115	117	119	120	121
60	103	107	109	111	111	112
80	95	98	101	103	104	105
100	90	94	97	99	101	102

EFFECT OF DENSITY ON SITE INDEX MEASUREMENT

Having determined a suitable measure of stocking, it was possible to compare heights of trees on paired plots on the basis of stocking. Designating the site index of a plot as Ip and the adjusted site index measured on an adjacent average-stocked control as Ic, the difference between the two site indices may be written as (Ic-Ip). Plotted over stocking, as is shown in figure 6, A, these site differences are greatest on the densest plots. A freehand straight line through the plotted points shows that as stockings approach 100 percent, differences in site indices approach zero.

Plotting the site differences on plot site index (Ip), shown in figure 6, B, indicates that differences are greatest on poor sites; as sites increase to site index 75, the differences approach zero, as the freehand straight line shows.

On the basis of some experiences with other species (Gaiser and Merz 1951; Turner 1943; Shirley and Zehngraff 1942) it might be expected that dense stocking on good sites would produce taller than average trees. In no instance was this condition observed or measured on paired plots in this study. For two reasons, stunting does not occur in dense ponderosa pine stands on good sites: first, very dense stands seldom become established on good sites because of the severe competition of associated vegetation; second, trees in these stands express dominance readily and thin themselves effectively before stagnation occurs.

It might further be expected that in stands having less than average stocking trees would be shorter than in average-stocked stands. MacKinney et al. (1937) found that in data from 68 plots of loblolly pine ranging in density from 80 to 120 percent height was reduced to some extent both by dense stocking and by open stocking. Heights were maximum at a stocking percentage of 100; the effects of stocking were not correlated with site.

No relationship between sparse stocking and height could be detected in ponderosa pine stands regardless of site quality. Open grown ponderosa pines apparently reach normal heights and at the same time produce larger diameters and crowns than trees in closed stands.

The effects of stocking on height of second-growth ponderosa pine in the Inland Empire can be summarized as follows:

1. Dominant heights are reduced on poor sites as stocking percentages increase above 100. If stocking percentages are below 100, heights are unaffected regardless of site.

2. Height stunting is intensified as site quality diminishes below site index 75. If site indices are above 75, heights are unaffected regardless of stocking.

To show these relationships of stocking and site quality on height, the factor (S-100)(75-Ip) was chosen. It increases as stockings increase above 100 and as plot site indices decrease below 75. If this factor, designated hereafter as Z, is taken as zero when either (S-100) or (75-Ip) is zero or negative, the desired conditions as enumerated above are met. To express the relationship of Z to differences in site indices of paired plots, the following formula was developed. (See Appendix II for details of the solution):

$$\log\left(\frac{Ic}{Ip}\right) = b_1 Z + b_2 Z^2 \qquad (2)$$

where $Z = (S-100)(75-Ip)$
and b_1 and b_2 = coefficients to be calculated.

Site indices for 52 plots (including the 24 used in the paired plot analysis) were adjusted by equation (2). A graph of this equation is shown in figure 13.

Having adjusted the site indices on all plots whose densities were above 100 and whose plot sites were below 75, all 207 plots could be pooled for a general site index analysis. Site curves were constructed on the basis of an equation that contained site index itself as one of the independent variables. Site was included to allow any differences in growth habits from site to site to be reflected in the shape of the curves. Reciprocal of age was a second variable; index age was taken at 100 years.

A second innovation in the site index equation was the inclusion of the variable Z to provide for site corrections for dense stands on poor sites. The site index equation with its calculated coefficients was determined to be:

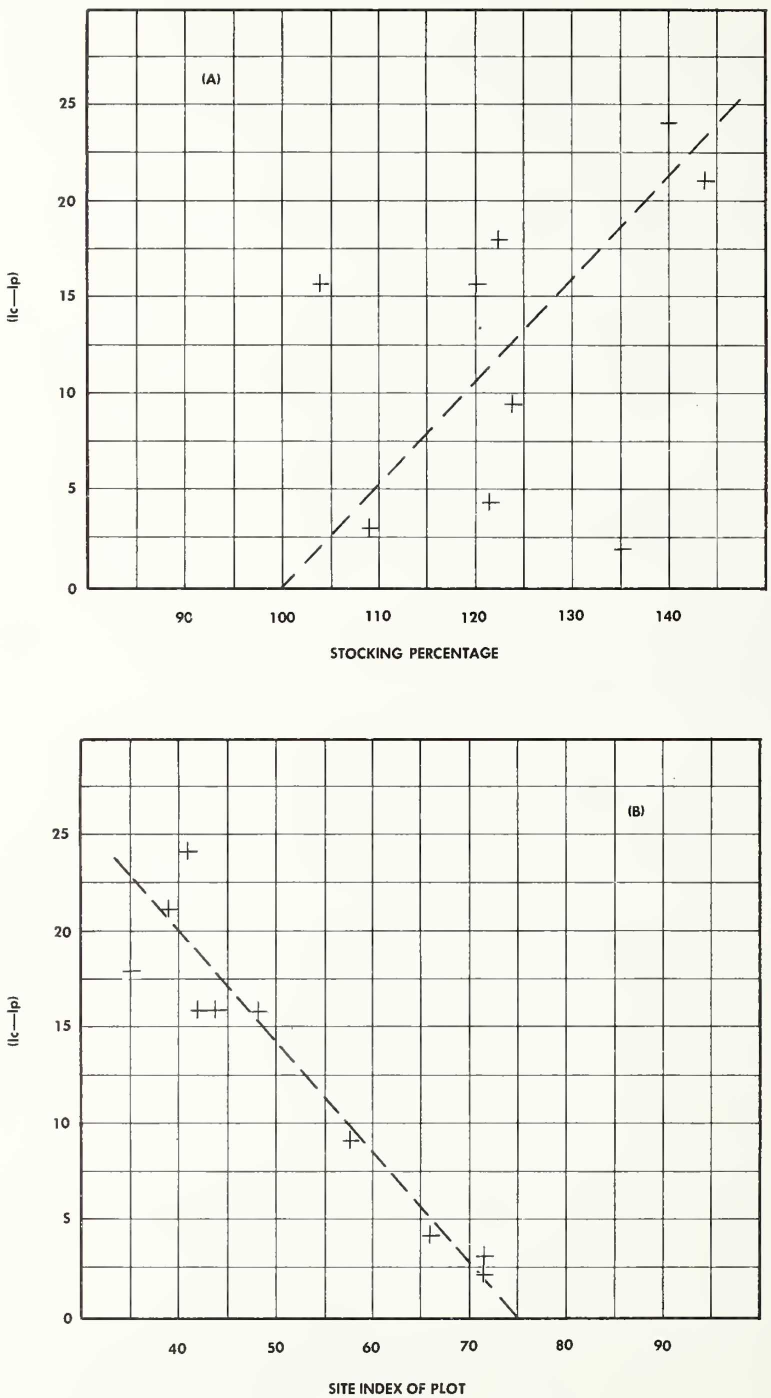

Figure 6.--A Differences in site indices between plots and adjacent average-stocked controls (Ic-Ip) are plotted on stocking percentage of the plot. They are greatest on the densest plots, and differences approach zero as stocking approaches 100. B The differences are plotted over the observed site indices of the plots. They are greatest on poor sites; as sites increase to site index 75, the differences approach zero.

$$\log H = \log I - 0.6437\left(\frac{100}{A} - 1\right) + 0.2432 \log I\left(\frac{100}{A} - 1\right)$$

$$- 0.000361(Z) + 0.00000012(Z^2) \qquad (3)$$

where H = dominant stand height
I = site index
A = dominant age
Z = (S-100)(75-Ip).

A more complete discussion of equation (3) is given in Appendix III.

Figure 7 A, B, and C presents sets of site curves for stocking percentages of 100 or less, 110, and 120.

The set of site curves for average stocking (100 or less) is comparable to the set from Meyer's bulletin. The two have been compared in figure 7 D. It will be noted that Meyer's curves are higher at the two extremes of age, but are lower in the central range. They are generally flatter in shape than the revised curves and vary less in shape from one site index to the next. These normal site curves have been criticized for use in the Inland Empire because they tend to underestimate site index in young stands. The curves from the present study will correct this fault to some extent and generally should be more suitable for use in the Inland Empire.

The early work of Behre (1938) should also be mentioned. His preliminary normal yield tables were constructed especially for northern Idaho and adjacent areas, and were used until superseded by Meyer's work. Behre's site curves differ from Meyer's by being lower on good sites and higher on poor sites for young ages. They vary considerably from the site curves of the present study.

The major difference between all former site curves and the present ones, of course, appears in sites below 75 for stocking percentages above 100. By using the appropriate set of curves, chosen on the basis of density of the stand in question, site index can be measured more precisely.

PREDICTING GROWTH IN PONDEROSA PINE STANDS

As a stand of timber develops, its change and growth can be described by such fundamental variables as age, dominant height, number of trees, basal area, and average d.b.h., as well as by such derived variables as volume in cubic feet and board feet. Predicting growth means predicting each of these variables for some future date.

The normal yield table method of growth prediction is simply to read from a table the present volume for the appropriate age and site; then, from the same table, read the volume at the future age and take the difference as volume growth. If a stand is understocked as determined by a comparison of observed basal area to normal, a percentage correction is applied to the initial and final volumes. The tables do not provide for stocking changes in understocked or overstocked stands. Herein lies the major weakness of normal yield tables for growth prediction. Stand densities do change, even in the span of one decade, to the extent that serious errors can be made in predicting growth of nonnormal stands if these changes are not considered. Understocked stands increase in density with time, and overstocked stands decrease. The changes are most rapid in young stands and in stands having density extremes.

As stocking approaches average for a particular species, site, and age, the changes of stocking with age are very slow. These characteristics of approach to normality have been measured in many forest types (Chaiken 1939; Briegleb 1942; Wellwood 1943; Watt 1950). Various methods have been used to adjust for changes in normality with the passing of time, most of which involve a percentage adjustment depending upon the age and initial stocking of the stand.

The process presented here was suggested by F.X. Schumacher, who applied the method to growth prediction of even-aged loblolly pine stands in an unpublished paper from Duke University, School of Forestry, (Schumacher and Coile, 1954).

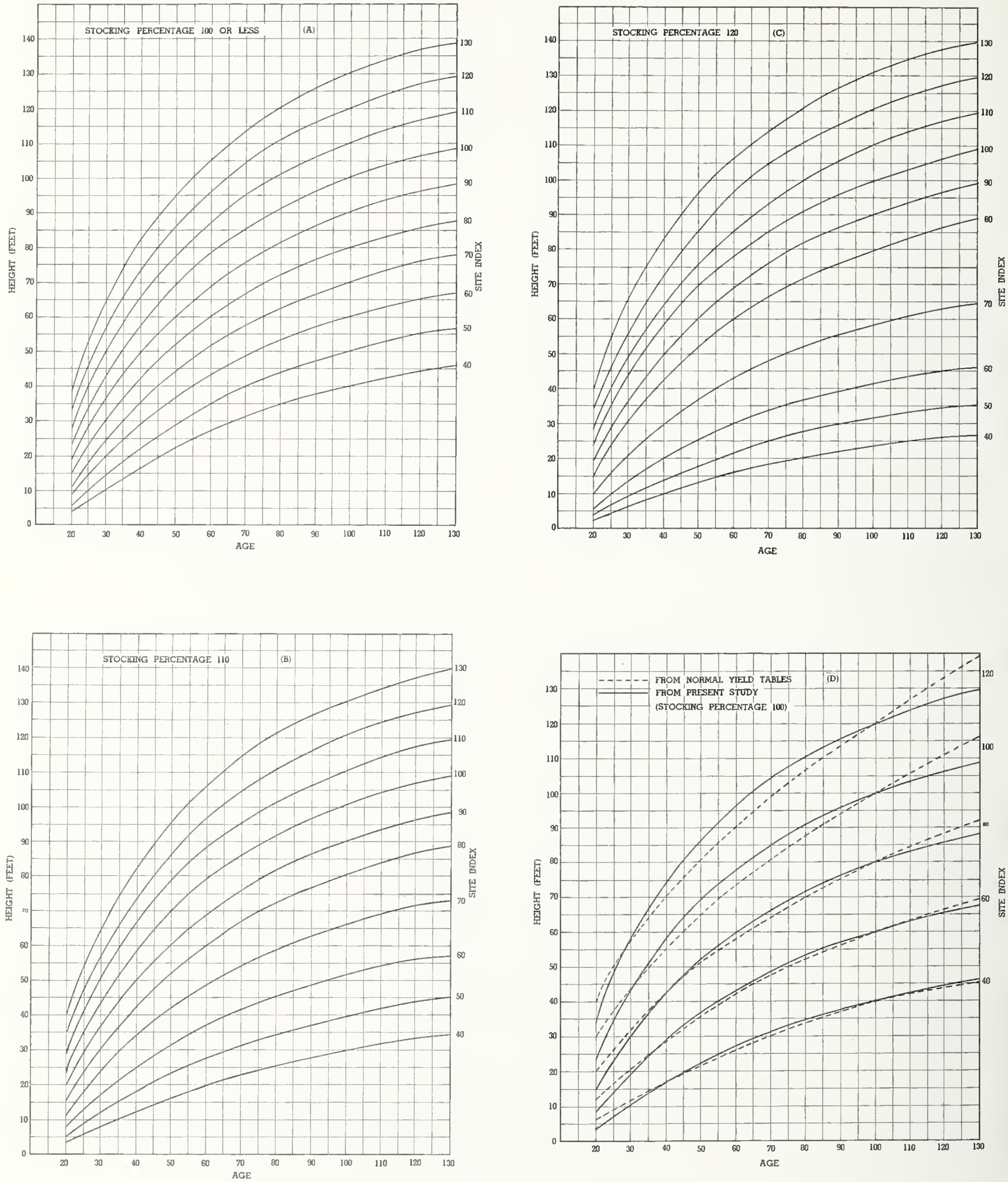

Figure 7.--Site curves: A for 100 percent or lower stocking, B for 110 percent stocking, C for 120 percent stocking, and D comparing Meyer's curves for normal yield study site with those of the present study for average stocking.

FUTURE STOCKING

For the growth prediction portion of this study, the stocking equation (1)[1/], was modified somewhat by eliminating the variable BH and recomputing the coefficients for the remaining variables. The revised form of the equation, shown below, was better adapted to predicting future basal area:

$$S = B\left[0.5663 - 0.2715\left(\frac{H}{A}\right) + 15.2858\left(\frac{1}{A}\right)\right]. \qquad (4)$$

The portion of equation (4) within the brackets is, as before, the stocking per square foot of basal area and may be designated s, as in the following:

$$s = 0.5663 - 0.2715\left(\frac{H}{A}\right) + 15.2858\left(\frac{1}{A}\right). \qquad (5)$$

In the prediction process, future age, A_1, of course, is readily obtained, and future height (H_1) is predicted from site curves. Therefore, s_1, or future stocking per square foot of basal area, can be computed from future age and height by equation (5).

An expression for predicting future stocking (S_1) that provides for rapid changes in stocking for young stands with extremes of density and that provides for slower changes as age advances and as stocking approaches 100, is the following:

$$\log S_1 = 2 + (\log S_0 - 2)\frac{A_0}{A_1}$$

where S_1 = future stocking
S_0 = present stocking
A_1 = future age
A_0 = present age.

This equation is derived and discussed in detail in Appendix IV, and plotted in figure 14 B.

From S_1 and s_1, future basal area can be determined from the relationship shown in equation (4) where

$$S_1 = B_1 s_1$$

$$\text{or } B_1 = \frac{S_1}{s_1}.$$

Figure 8 shows basal area plotted on age for site indices 50 to 100. Each graph contains curves of basal area on age for various initial basal areas at indicator-age 30. Knowing present basal area, age, and site, future basal area can be read for ages up to 120 years.

FUTURE NUMBER OF TREES

The number of trees in a stand is a function of the age, the basal area, and the height. To express this relationship in a linear equation, the logarithmic transformation of height and basal area and the reciprocal transformation of age are required. The equation, derived in detail in Appendix V, is:

$$\log N = b_0 + b_1(\log H) + b_2\left(\frac{1}{A}\right) + b_3(\log B) \qquad (6)$$

where N = number of trees per acre
H = dominant height in feet
A = age in years
B = basal area per acre in square feet
b_0 to b_3 = coefficients to be computed.

1/ p. 10.

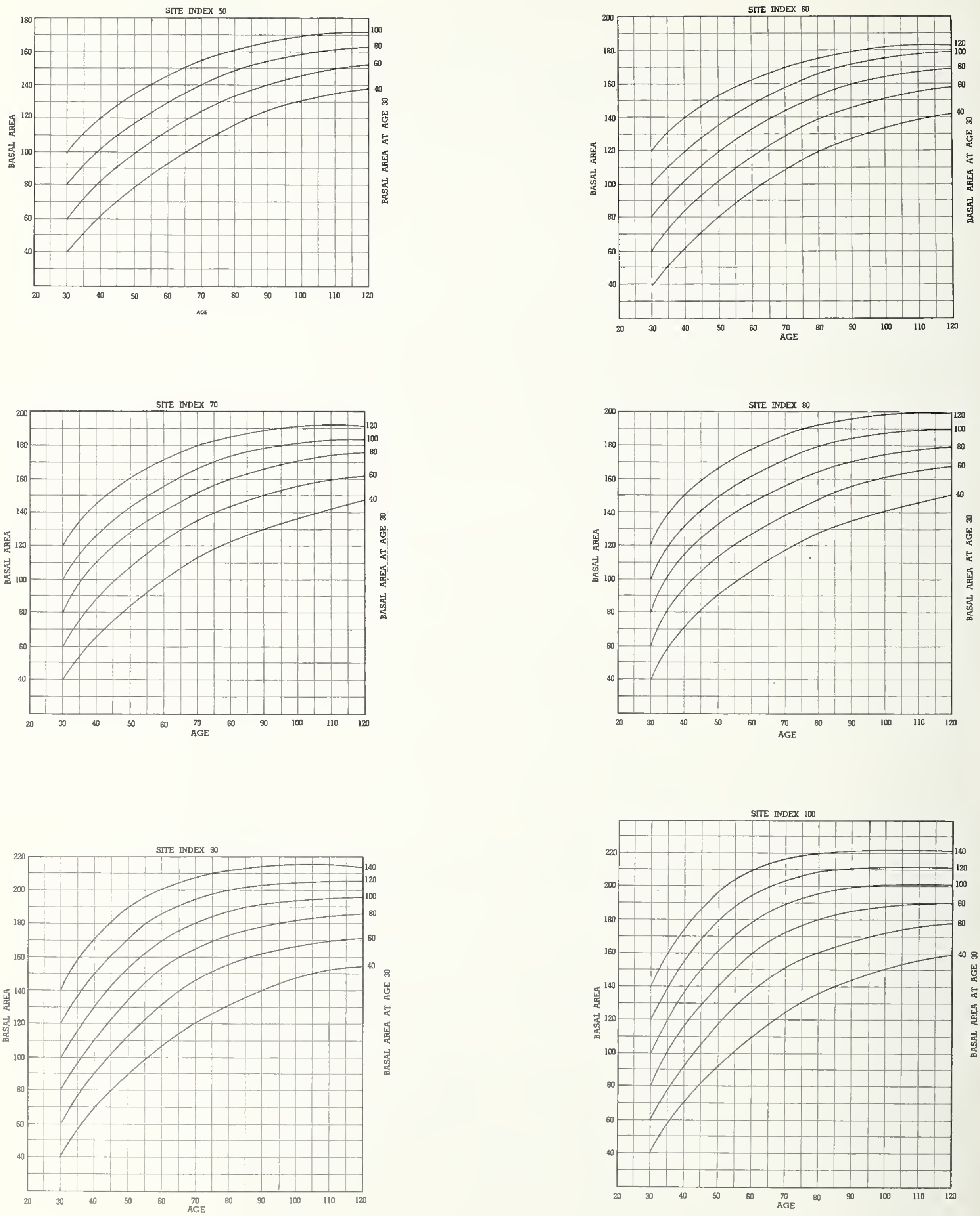

Figure 8.--Curves of basal area on age for various initial basal areas at age 30 and for site indices 50, 60, 70, 80, 90, and 100.

Using subscripts 1 and 0 to designate future and present, respectively, as before, the difference between future and present log N can be written,

$$\log N_1 - \log N_0 = \log\left(\frac{N_1}{N_0}\right)$$

$$= b_1 \log\left(\frac{H_1}{H_0}\right) + b_2\left(\frac{1}{A_1} - \frac{1}{A_0}\right) + b_3\left(\log\frac{B_1}{B_0}\right)$$

in which the constant, b_0, drops out.

The coefficients for equation (6) were computed from the basic data from the 207 plots of this study; from these coefficients the completed form of the above equation was determined as follows:

$$\log\left(\frac{N_1}{N_0}\right) = -2.6078\left(\log\frac{H_1}{H_0}\right) - 11.2150\left(\frac{1}{A_1} - \frac{1}{A_0}\right) + 1.4579\left(\log\frac{B_1}{B_0}\right). \qquad (7)$$

It is apparent that the ratio of future number of trees to present number is dependent upon height, age, and basal area, the values of which can be predicted. A graphic presentation of equation (7) appears in figure 9 for site indices from 50 to 100. Separate curves are drawn for various initial basal areas over a range of age from 30 to 120 years.

If a stand on site index 60, for example, contains 500 trees with 60 square feet of basal area at indicator-age 30, then at age 50, 56 percent of these, or 280 trees, should still survive. The graphs may be used for any initial age. For example, the 50-year-old stand with indicator-age basal area of 60 has 56-percent survival, and at age 70 has 43 percent. Then, 43/56 or 77 percent of the 280 trees at age 50 should survive at age 70.

Having estimated future basal area and number of trees per acre, the average basal area per tree of the future stand can readily be determined. By means of a standard basal area table or, more conveniently from the graph in figure 10, average d.b.h. in inches can be read for any average basal area in square feet.

FUTURE CUBIC-FOOT VOLUME

Total cubic-foot volume of a tree, inside bark, is basic to the determination of other measures of volume, such as the board-foot or the cord. Cubic-foot volume on an acre basis is a function of the square of the average stand diameter, the height of the dominant stand, and the number of trees on the acre. In equation form it is:

$$\log V = b_0 + b_1(\log D^2) + b_2(\log H) + b_3(\log N)$$

where V = cubic volume per acre
D = diameter of tree of average basal area in inches
H = dominant height in feet
N = number of trees per acre
b_0 to b_3 = coefficients to be computed.

Based on the data from the 207 plots of this study, the volumes of which were computed from a table by Meyer (1938), the computed equation is:

$$\log V = -2.6950 + 1.0203(\log D^2) + 0.9704(\log H) + 0.9996(\log N). \qquad (8)$$

The coefficient of log N was not a significant departure from 1. (See Appendix VI, for the details of developing this equation.) If height and average diameter are constant, cubic volume of a stand is in direct proportion to the number of trees. Treating this coefficient as unity, the average volume per tree, v, can be expressed as:

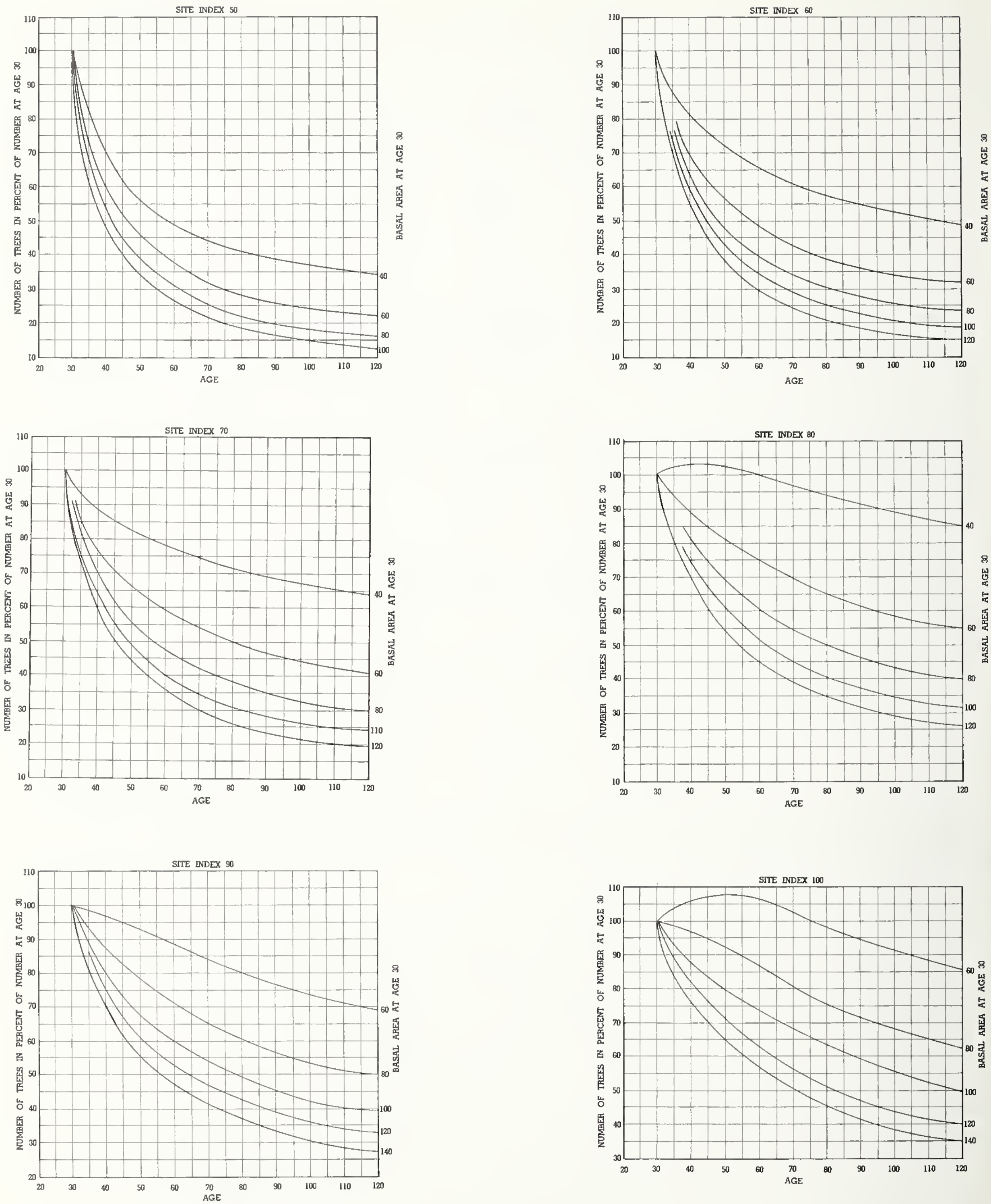

Figure 9.--Curves showing change in relative number of trees for various ages and initial basal areas and for site indices 50, 60, 70, 80, 90, and 100.

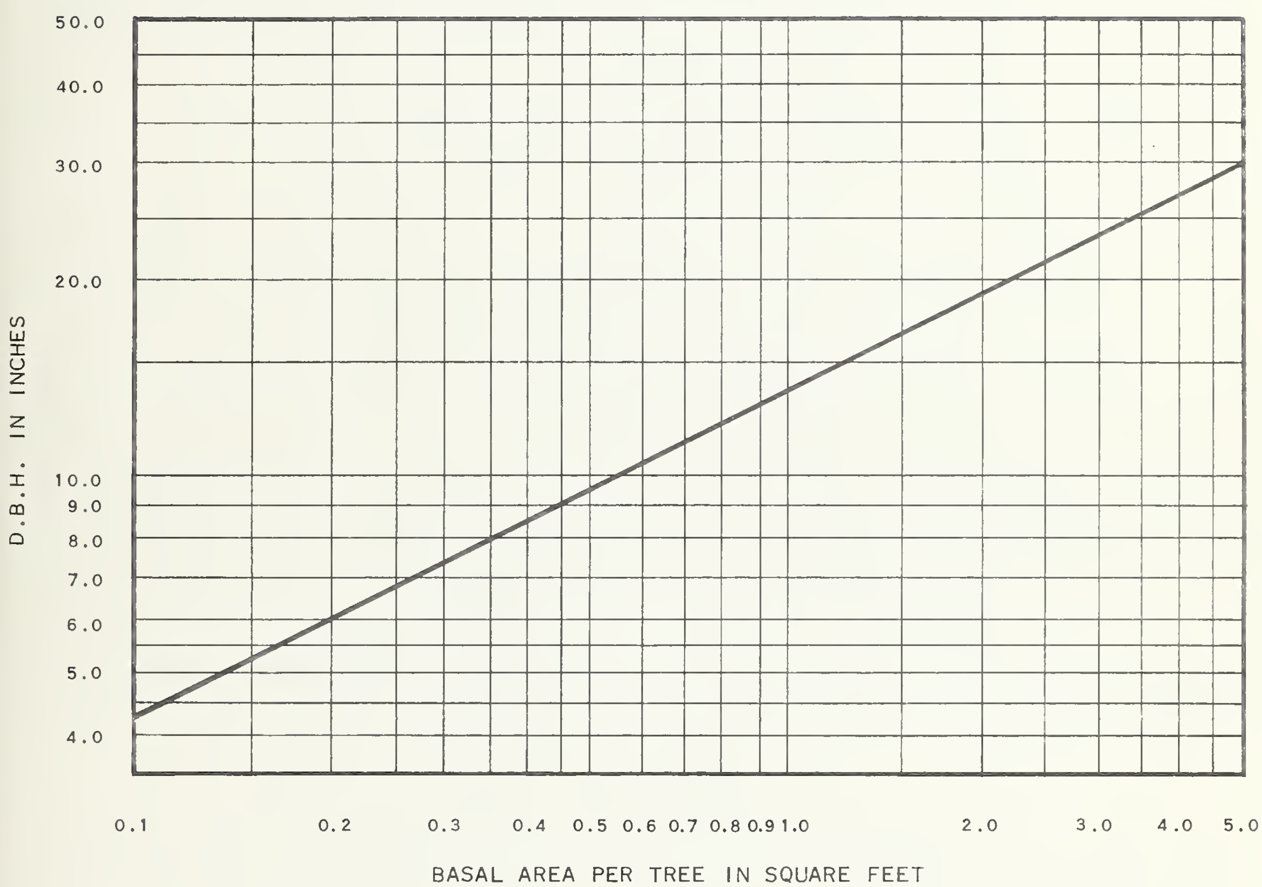

Figure 10.--Average stand d.b.h. in inches in terms of average basal area per tree in square feet.

$$\log V - \log N = \log\left(\frac{V}{N}\right) = \log v \tag{9}$$

$$= -2.6950 + 1.0203(\log D^2) + 0.9704(\log H).$$

Equation (9) is shown graphically in figure 11 for various average diameters and for various heights. Having predicted future average diameter and height, the average cubic-foot volume per tree can be read from the graph in figure 11. Multiplying this volume by the future number of trees gives an estimate of future cubic-foot volume per acre.

FUTURE BOARD-FOOT VOLUME

Other measures can be derived from cubic-foot volume per acre. In the present economy of the Inland Empire, the only other unit of measure needed is volume in board feet. Since there is no market for pulpwood, second-growth pine saw logs are often cut to diameters as small as 10 inches. Therefore, converting factors have been computed for changing total cubic-foot volume per acre to board-foot volume per acre in trees 10 inches d.b.h. and larger.

Board-foot volumes were computed by the International 1/4-inch rule to a variable top from a volume table developed for use by the Forest Survey in the Intermountain region.

The ratio of board feet in trees 10 inches d.b.h. and larger to cubic feet of all trees can be expressed linearly as

$$\log R = b_0 + b_1\left(\frac{1}{D^2}\right) + b_2\left(\frac{H}{D^2}\right)$$

where R = board-foot/cubic-foot ratio
D = diameter of tree of average basal area
H = dominant height
b_0 to b_2 = coefficients to be computed.

This equation is developed in Appendix VII.

Based on actual ratios from 121 plots (the other 86 plots had no trees as large as 10 inches), the computed equation, with two of the variables coded for convenience, is:

$$\log(R \times 100) = 2.7256 - 0.6501\left(\frac{100}{D^2}\right) + 0.6730\left(\frac{H}{D^2}\right). \tag{10}$$

Equation (10) is shown graphically in figure 12 for various average diameters and dominant heights. It should be kept in mind that these ratios are not for trees having these dimensions, but rather are for stands whose average diameters and dominant heights are those shown.

YIELD OF AVERAGE-STOCKED STANDS

It is interesting to compare the yields of average-stocked stands (stocking percentage 100) determined in this study with Meyer's (1938) normal yield tables. Table 2 shows dominant heights and average-stand diameters by age classes and site classes for all trees 0.6 inch d.b.h. and larger. On an acre basis the table also shows basal areas, number of trees, cubic volume of all trees above 0.6 inch d.b.h., and board-foot volume in trees 10 inches d.b.h. and larger.

Table 2 can be used to predict yields if the stands concerned happen to be 100 percent stocked. For predicting growth in under- or overstocked stands, the method described in the previous section of this report should be used.

In general, the basal areas shown in table 2 are smaller than Meyer's for younger ages but continue to rise slowly for ages up to 120, whereas Meyer's basal areas level off at as early as 60 years on poor sites. In stands above site index 90 Meyer's basal areas are higher throughout the range of age shown in table 2.

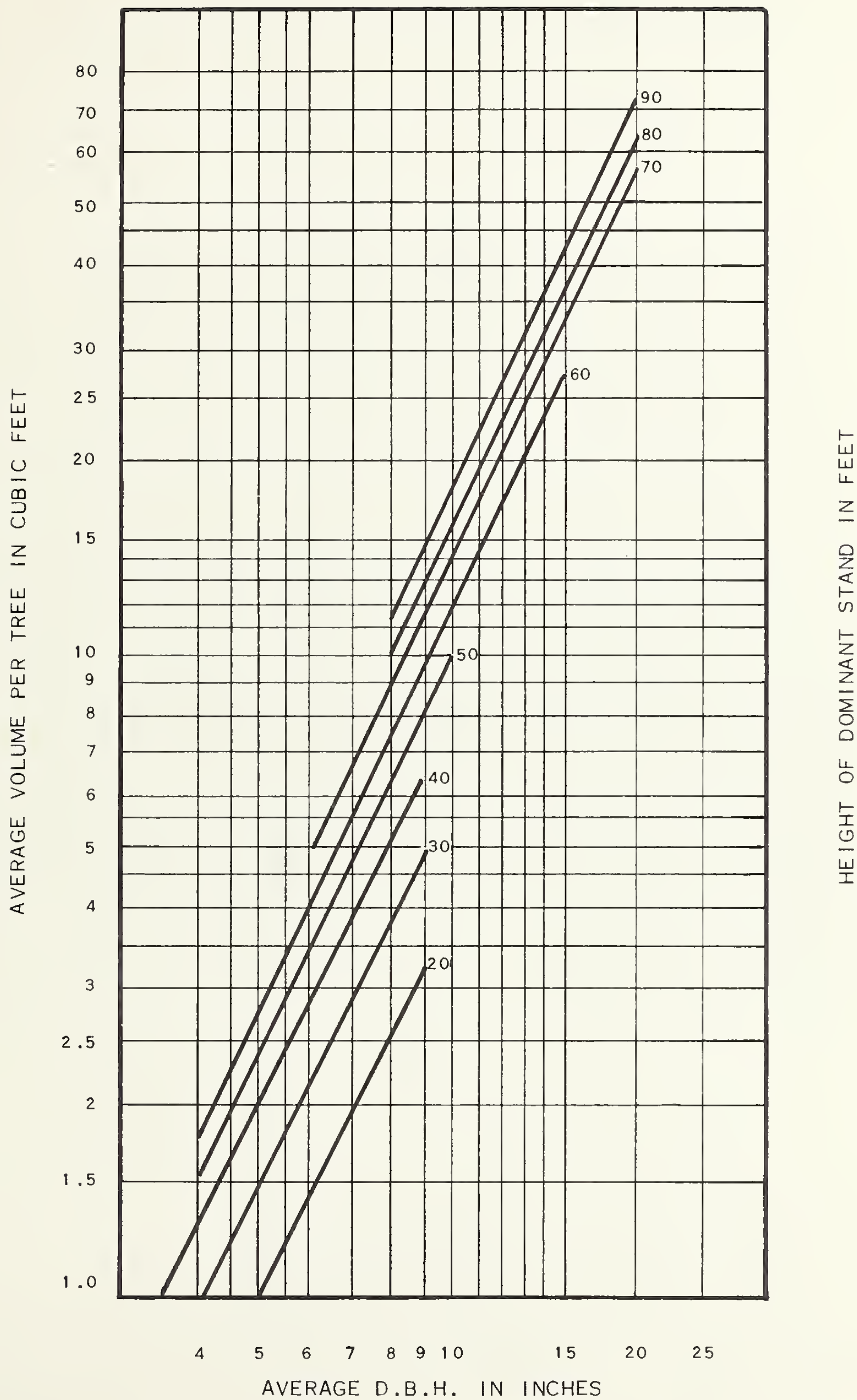

Figure 11.--Average volume per tree in cubic feet for various average stand diameters and dominant heights.

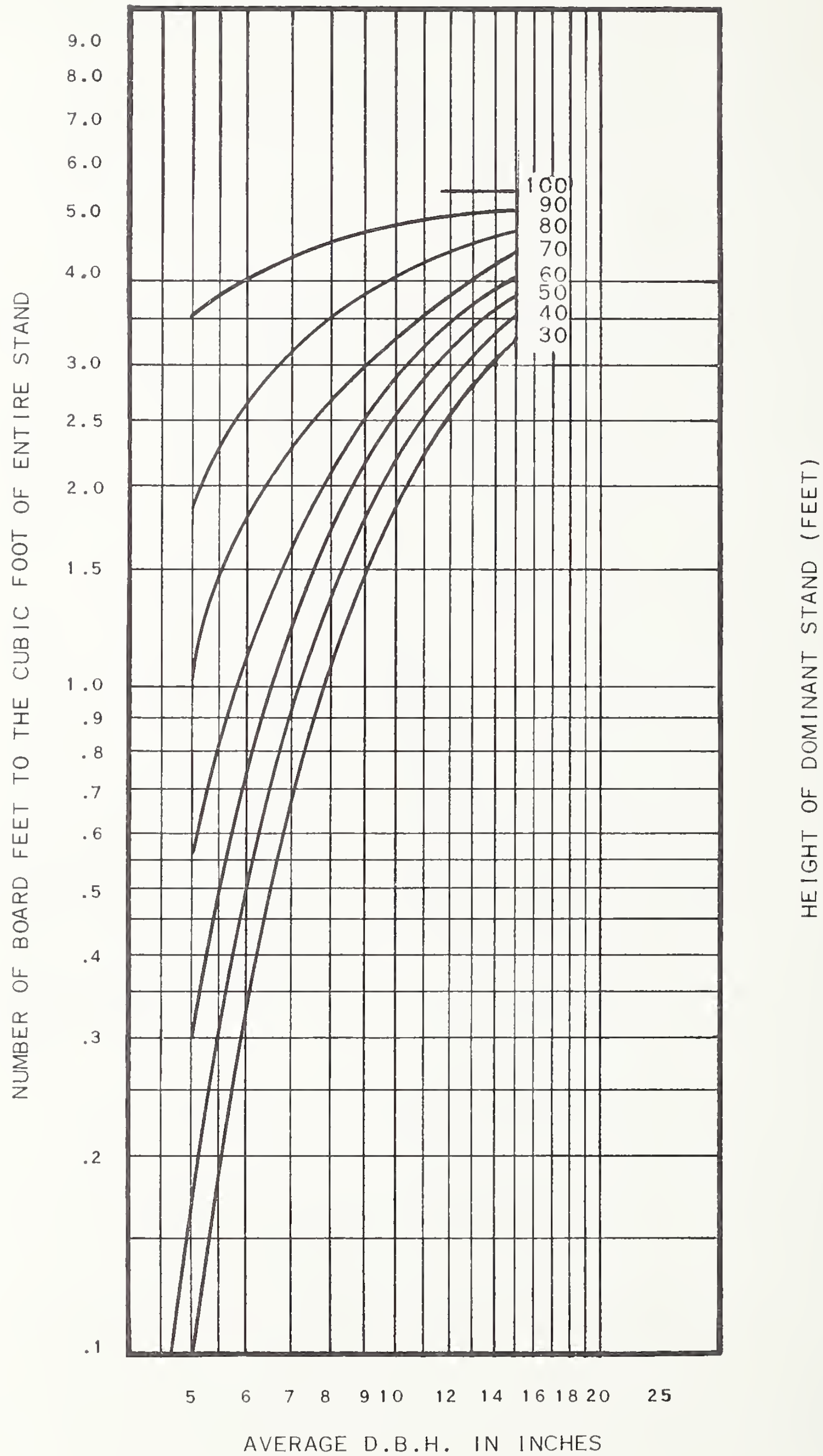

Figure 12.--Ratios of board feet in trees 10 inches d.b.h. and larger to total cubic feet in all trees 0.6 inch and larger for various average stand diameters and dominant heights.

Table 2.--Yields per acre of average-stocked second-growth ponderosa pine stands showing height, basal area, number of trees, average diameter, cubic-foot volume, and board-foot volume for ages 40 to 120 and site indices 50 to 110. All trees 0.6 inch d.b.h. and larger are included

Age (years)	Height of dominant stand	Basal area per acre	Trees per acre	Average d. b. h.	Total volume[1/]	Volume in trees 10 in. & over[2/]
	Feet	Sq. ft.	Number	Inches	Cu. ft.	Bd. ft.
			Site Index 50			
40	22	125	3,334	2.6	950	0
60	35	151	1,621	4.1	1,830	0
80	44	164	1,122	5.2	2,580	720
100	50	171	911	5.9	3,060	2,080
120	55	176	775	6.5	3,480	4,000
			Site Index 60			
40	29	132	1,756	3.7	1,340	0
60	43	160	1,031	5.3	2,350	700
80	53	173	747	6.5	3,240	2,400
100	60	180	611	7.3	3,790	6,800
120	65	183	531	7.9	4,180	10,100
			Site Index 70			
40	35	141	1,184	4.7	1,770	0
60	52	170	686	6.7	3,110	3,600
80	63	183	517	8.1	4,150	9,960
100	70	189	439	8.9	4,740	14,900
120	76	191	376	9.6	5,120	18,900
			Site Index 80			
40	42	151	814	5.8	2,230	1,000
60	60	182	522	8.0	3,900	8,580
80	72	195	400	9.5	5,420	18,860
100	80	199	334	10.4	6,030	25,300
120	86	200	291	11.2	6,560	30,370
			Site Index 90			
40	50	164	582	7.2	2,940	3,800
60	69	197	407	9.4	4,840	15,970
80	82	209	315	11.0	6,100	26,840
100	90	211	267	12.0	6,760	33,660
120	96	210	234	12.8	7,200	38,160
			Site Index 100			
40	58	180	453	8.5	3,700	10,540
60	78	214	333	10.9	6,030	25,330
80	91	223	264	12.4	7,230	36,300
100	100	223	220	13.6	7,970	43,840
120	106	220	194	14.4	8,360	48,070
			Site Index 110			
40	66	200	377	9.9	4,770	15,740
60	88	235	279	12.4	7,390	36,200
80	101	241	225	14.0	8,730	48,000
100	110	238	189	15.2	9,420	55,600
120	116	232	166	16.0	9,670	59,000

1/ Total cubic-foot volume inside bark in trees 0.6 inch d.b.h. and larger. Volumes per tree taken from Meyer's (1938) bulletin.

2/ Board-foot volumes in trees 10 inches d.b.h. and larger by the International 1/4-inch rule to a variable top.

The number of trees per acre is lower in table 2 for young ages but higher in old ages than in Meyer's tables. These relationships of basal area and number of trees result in average diameters being slightly higher in table 2 for young stands and lower for old stands than those shown in the normal yield table. Cubic-foot volumes are generally somewhat lower for comparable ages and sites than in the normal yield tables. This could be expected because the 100 percent stocking used in this study is still understocked according to the normal yield tables, as table 1 shows.

Values in table 2 have only limited use in growth prediction, but they are of interest because they show a comparison of Inland Empire stands with the so-called normal stands of the ponderosa pine type covered in Meyer's interregional study.

SUMMARY

Dense stocking in second-growth ponderosa pine stands on poor sites in the Inland Empire causes stunting of heights sufficient to impair site index determination by the usual methods. The present study was conducted to measure the extent of this stagnation and to develop adjusted site index curves for use in overstocked stands. A second objective of the study was to present a method for growth prediction, especially suited to understocked stands, which accounts for changes in stocking with time.

The effects of stocking on heights, as reported by other investigators for several forest types, are reviewed; generally, height retardation due to stocking is correlated with site and with the inherent ability of the species to express dominance. In a few instances, dense stocking results in taller than average trees, particularly on good sites; but more commonly, if stocking has any effect at all, it retards height growth.

Data for this study were taken from 207 temporary plots well distributed geographically over the Inland Empire and covering a wide range of sites, stand densities, and ages up to 125 years.

Stocking was evaluated by an equation based on basal area, age, and height, giving a stocking percentage for each plot; a stocking of 100 percent represents the average for all plots of the study. This average stocking is somewhat lower than the normal yield table stocking shown in Meyer's (1938) bulletin.

Height retardation was found to increase as stocking increased above 100 percent and to increase as site indices decreased below site index 75. These effects were measured on paired plots where abrupt changes in stocking were associated with marked differences in stand heights on areas of uniform site insofar as soil, aspect, and surface vegetation could determine.

Adjusted site index curves are presented for stocking percentages 100 or less, 110, and 120. In addition to the differences resulting from the introduction of stocking, the curves differ from the normal yield site curves by being lower for younger and older ages, but higher in the central range of ages. They are believed to fit the conditions of the Inland Empire better than the interregional normal yield curves.

For purposes of predicting future yields of second-growth ponderosa pine stands, graphs are presented, for various site indices, showing basal area by age for different initial basal areas. Graphs showing the relationship of present and future numbers of trees for various initial basal areas are also presented for a range of site indices.

From an estimate of future basal area and number of trees, the forester can determine from appropriate graphs the future average-stand diameter, average volume per tree, and volume per acre. Finally, a method of converting total cubic-foot volume per acre to board-foot volume in trees 10 inches d.b.h. and larger, is presented graphically.

A table of yields for average-stocked stands shows heights, average diameters, basal areas, number of trees, cubic-foot volumes, and board-foot volumes.

The details of the analytical methods are described in the Appendix.

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A P P E N D I X

I. STOCKING PERCENTAGE

Tree-area ratio, as presented by Chisman and Schumacher (1940), expressed the ground area, Y, of a tree in the forest as a function of its d.b.h., d, such that

$$Y = b_0 + b_1 d + b_2 d^2.$$

The sum of the areas occupied by the n trees of a plot is, then,

$$\sum^{n}(Y) = b_0(n) + b_1\sum^{n}(d) + b_2\sum^{n}(d^2), \tag{11}$$

where $\sum^{n}$ denotes sum of the n values. The observation equations for individual trees cannot be written, because a single tree occupies an unknown area in the forest. But the area that a group of trees occupies, on a one-fifth-acre plot, can be expressed as in equation (1) in terms of the number of trees, sum-of-diameters, and sum-of-diameters-squared. Thus, a group of plots supplies observations for computing the coefficients for the equation from which the area occupied by individual trees may be determined. If plot areas are constant and expressed as unity (or more conveniently as 100), the equation gives relative stocking for any particular set of plot data.

Experience of others with this equation has shown that sum-of-diameters usually is not significant. Basal area can, of course, be substituted for sum-of-diameters-squared because of the constant relationship,

$$B = \Sigma\,(d^2)0.005454.$$

Other stand variables also affect tree area to a considerable degree. Two important ones to consider are age and height, which can be included by beginning with the equation,

$$S = b_1 N + b_2 B,$$

where S is plot area (conveniently taken as 100), N is number of trees per plot, and B is basal area. The coefficient of B can be considered a function of height and the reciprocal of age; that is,

$$b_2 = a_0 + a_1(H) + a_2\left(\frac{H}{A}\right) + a_3\left(\frac{1}{A}\right).$$

Substituting this value of b_2 into the equation above gives

$$S = b_1 N + \left[a_0 + a_1(H) + a_2\left(\frac{H}{A}\right) + a_3\left(\frac{1}{A}\right)\right] B$$

or

$$S = b_1 N + b_2(B) + b_3(BH) + b_4\left(\frac{BH}{A}\right) + b_5\left(\frac{B}{A}\right). \tag{12}$$

Solution of this equation by the method of least squares for a group of plot data of uniform area, 100, gives stocking directly in percent. It was solved in this study using data from 207 plots; the contribution of each of the variables is shown in table 3. The added effect of N, although significant, was least important of any of the variables. Because the presence of N in the stocking equation would render it difficult to use in the growth prediction phase of the study, N was dropped, and the coefficients for the remaining variables computed independently. The completed equation appears on page 10.

Table 3.--Analysis of variance showing the contribution of each variable in the stocking equation,

$$S = b_1(N) + b_2(B) + b_3(BH) + b_4\left(\frac{BH}{A}\right) + b_5\left(\frac{B}{A}\right).$$

Source of variation	Degrees of freedom	Sum of squares	Mean square
Regression on B	1	1,805,664	1,805,664*
Added effect of BH	1	54,592	54,592*
Added effect of B/A	1	7,259	7,259*
Added effect of BH/A	1	31,039	31,039*
Added effect of N	1	5,848	845
Residuals	202	165,598	
Total	207	2,070,000	

* Significant beyond the 1-percent level.

In using the stocking equation to predict future basal area, where basal area becomes the dependent variable, it was necessary to drop BH from the equation. The presence of this variable gave unrealistic values of basal area for advanced ages. Dropping BH and recomputing the coefficients for the remaining variables gave a more satisfactory equation and at the same time made very little difference in the stocking percentages computed. The contributions of the variables in the altered equation are shown in table 4.

Eliminating the variable BH increased the residual mean square from 845 to 898, or increased the uncorrelated variance from 8.28 to 8.37 percent.

Table 4.--Analysis of variance showing the contribution of each variable in the stocking equation,

$$S = b_2B + b_4\left(\frac{BH}{A}\right) + b_5\left(\frac{B}{A}\right).$$

Source of variation	Degrees of freedom	Sum of squares	Mean square
Regression on B	1	1,805,664	1,805,664*
Added effect of BH/A	1	41,199	41,199*
Added effect of B/A	1	39,890	39,890*
Residuals	204	183,247	898
Total	207	2,070,000	

* Significant beyond the 1-percent level.

II. PAIRED PLOT ANALYSIS

The factor Z, i.e. (S-100)(75-Ip), is a variable that was used to measure the effect of site index discrepancies between paired plots; one plot had dense stocking, but the other (the control) had only average stocking. Negative values of (S-100) and of (75-Ip) were considered zero.

Since height and site indices plot as straight lines in the logarithmic transformation, the difference between the site indices of paired plots was written as

$$\log Ic - \log Ip = \log\left(\frac{Ic}{Ip}\right)$$

where Ic = site index of control
Ip = site index of plot.

Expressing the logarithm of the ratios in the first and second power of Z and fitting the equation to the data from the 24 sets of paired plots gave the following equation:

$$\log Ic/Ip = 0.000269(Z) - 0.000093\left(\frac{Z^2}{1000}\right) \tag{13}$$

The analysis of variance is shown in table 5, and the equation is plotted in figure 13.

Table 5.--Analysis of variance showing the contribution of the two variables in the equation

$$\log Ic/Ip = b_1 Z + b_2 Z^2.$$

Source of variation	Degrees of freedom	Sum of squares	Mean square
Regression on Z	1	0.127912	0.127912*
Added effect of Z^2	1	.006349	.006349**
Residuals	22	.029686	.001349
Total	24	0.163947	

* Significant beyond the 1-percent level.
** Significant at the 5-percent level.

III. THE SITE INDEX EQUATION

Starting with the basic equation, in which logarithm of height is a function of the reciprocal of age,

$$\log H = a + b\left(\frac{1}{A}\right). \tag{14}$$

Introducing log I as an independent variable to allow for differences in the growth curves from site to site, the coefficients from equation (14) become

$$a = a_0 + a_1 (\log I)$$

and

$$b = b_0 + b_1 (\log I).$$

Substituting:

$$\log H = a_0 + a_1(\log I) + \left[b_0 + b_1(\log I)\right] \left(\frac{1}{A}\right). \tag{15}$$

Imposing the site index concept, in which log H equals log I at index age 100, gives

$$\log I = a_0 + a_1(\log I) + \left[b_0 + b_1(\log I)\right] \frac{1}{100}$$

or

$$a_0 = \log I - a_1(\log I) - \frac{b_0}{100} - b_1\frac{\log I}{100}.$$

Substituting this value of a_0 in equation (15) gives

$$\log H = \log I - a_1(\log I) - \frac{b_0}{100} - b_1\frac{\log I}{100}$$

$$+ a_1(\log I) + \frac{b_0}{A} + b_1\frac{\log I}{A} .$$

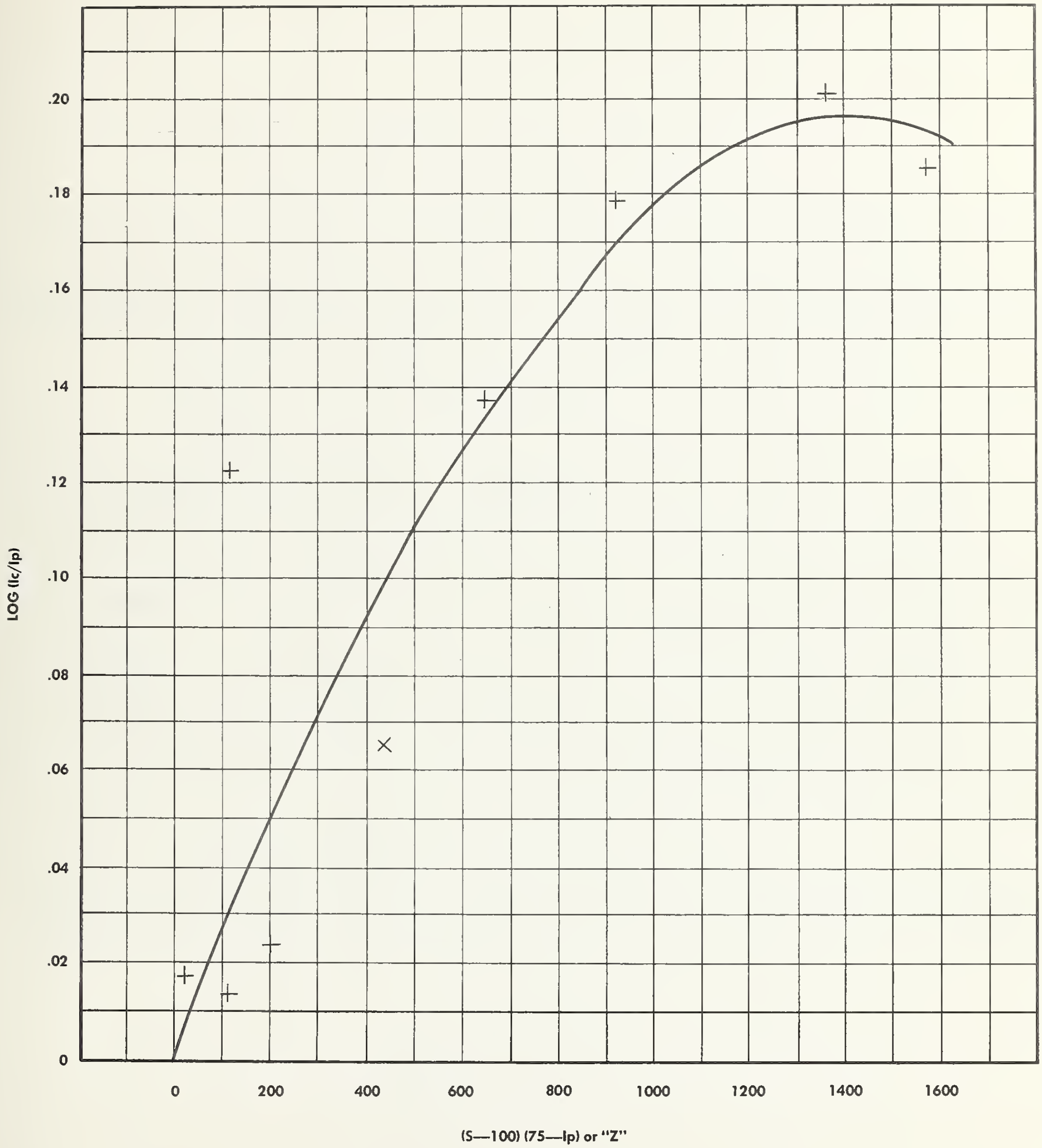


Figure 13.--Graph of the logarithm of the ratio of control site index to plot site index over the factor Z.

Collecting terms,

$$\log H = \log I + b_0\left(\frac{1}{A} - .01\right) + b_1 \log I\left(\frac{1}{A} - .01\right)$$

In the same manner, Z can be introduced to give the equation:

$$\log H = \log I + b_0\left(\frac{1}{A} - .01\right) + b_1 \log I\left(\frac{1}{A} - .01\right)$$
$$+ b_2 Z\left(\frac{1}{A} - .01\right) + b_3 Z \log I\left(\frac{1}{A} - .01\right) + b_4 Z + b_5 Z^2.$$

The separate independent variables Z and Z^2 were added to allow site indices less than 75 to fall below the site index points for age 100.

In calculating coefficients for this equation, the variables in which Z appears with the others were nonsignificant, and the equation took the form shown on page 13. The contributions of the variables in the regression analysis are shown in table 6.

Table 6.--Analysis of variance showing the contribution of each variable in the equation,

$$\log H - \log I = b_0\left(\frac{1}{A} - .01\right) + b_1 \log I\left(\frac{1}{A} - .01\right) + b_4 Z + b_5 Z^2.$$

Source of variation	Degrees of freedom	Sum of squares	Mean squares
Regression on $\left(\frac{1}{A} - .01\right)$	1	14.0512	14.0512*
Added effect of Z	1	.7520	.7520*
Added effect of log $I\left(\frac{1}{A} - .01\right)$	1	.2261	.2261*
Added effect of Z^2	1	.0353	.0353*
Residuals	203	.1965	.0010
Total	207	15.2611	

* Significant beyond the 1-percent level.

IV. AN EQUATION FOR FUTURE STOCKING

Figure 14, A shows the change in stocking with advancing age, the understocked plots increasing in density and the overstocked plots decreasing. In figure 14, B the two outside curves of figure 14, A are shown as straight lines when stocking is transformed to logarithms and age to its reciprocal. In this figure, S is stocking percentage and A is age; with subscripts 0 for present and 1 for future, then,

$$\text{length } PQ = \frac{1}{A_0}$$

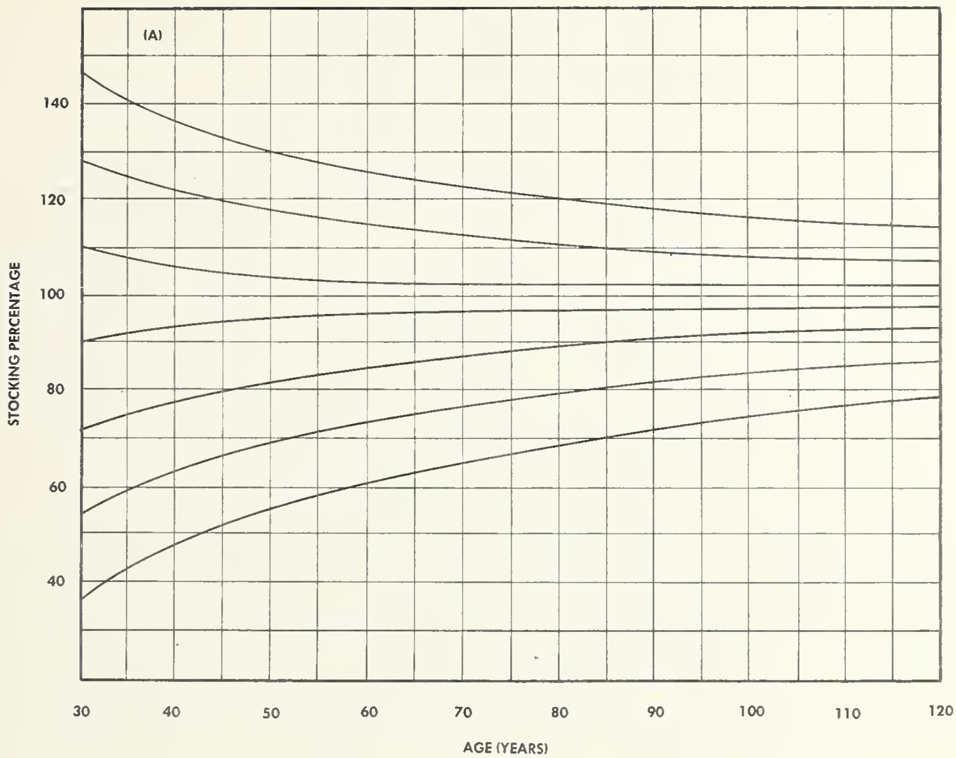
and

$$\text{length } QP_2 = \log S_0 - 2$$

$$\text{slope of line } PP_2 = \left(\frac{\log S_0 - 2}{\frac{1}{A_0}}\right)$$

$$\text{length } P_1Q_2 = \frac{1}{A_0} - \frac{1}{A_1}$$

$$Q_2P_2 = \left(\frac{\log S_0 - 2}{\frac{1}{A_0}}\right)\left(\frac{1}{A_0} - \frac{1}{A_1}\right) = (\log S_0 - 2)\left(1 - \frac{A_0}{A_1}\right).$$



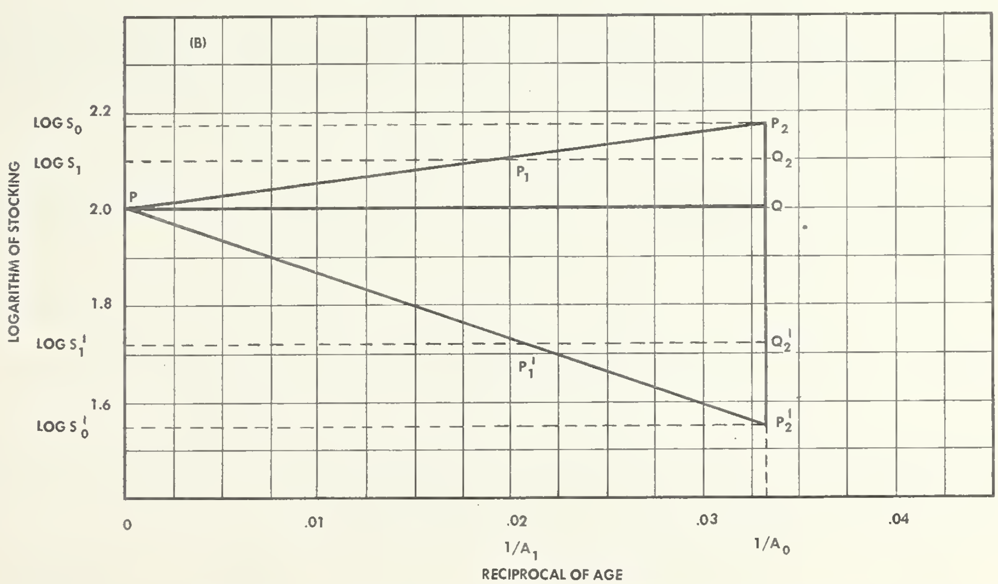


Figure 14.--A, Change in stocking (approach to normal) is shown for ages 30 to 120 and for various initial densities. B, A graphic explanation of the change in stocking equation

$$\log S_1 = 2 + (\log S_0 - 2)\left(\frac{A_0}{A_1}\right).$$

Then $$QQ_2 = (\log S_1 - 2) = (\log S_0 - 2) - (\log S_0 - 2)\left(1 - \frac{A_0}{A_1}\right)$$

or $$(\log S_1 - 2) = (\log S_0 - 2)\left(\frac{A_0}{A_1}\right)$$

and $$\log S_1 = (\log S_0 - 2)\left(\frac{A_0}{A_1}\right) + 2 \ . \tag{16}$$

Similarly,

$$\text{length } QP_2' = (2 - \log S_0') \ ;$$

$$\text{slope of line } PP_2' = \frac{(2 - \log S_0')}{\frac{1}{A_0}} \ .$$

$$Q_2'P_2' = \frac{(2 - \log S_0')}{\frac{1}{A_0}}\left(\frac{1}{A_0} - \frac{1}{A_1}\right) = (2 - \log S_0')\left(1 - \frac{A_0}{A_1}\right).$$

$$QQ_2' = (2 - \log S_1') = (2 - \log S_0') - (2 - \log S_0')\left(1 - \frac{A_0}{A_1}\right) .$$

Then $$(2 - \log S_1') = (2 - \log S_0')\left(\frac{A_0}{A_1}\right)$$

or $$- \log S_1' = -2 + (2 - \log S_0')\left(\frac{A_0}{A_1}\right)$$

and $$\log S_1' = (\log S_0' - 2)\left(\frac{A_0}{A_1}\right) + 2. \tag{17}$$

Equations 16 and 17 are appropriate for determining future stocking for both overstocked and understocked stands.

V. NUMBER OF TREES

Number of trees per acre can be expressed linearly by the logarithmic transformation of the variables, number of trees (N), dominant height (H), and basal area (B), and by the reciprocal of age $\left(\frac{1}{A}\right)$, in the form

$$\log N = b_0 + b_1 (\log H) + b_2\left(\frac{100}{A}\right) + b_3 (\log B) \ . \tag{18}$$

The computed coefficients for this equation appear on page 17. Its analysis of variance is shown in table 7.

Equation (18) can, of course, be used for present number of trees as well as future number by substituting appropriate values. If subscript 0 indicates present and subscript 1 indicates future values of the variables, the difference between future and present log N is

$$\log N_1 - \log N_0 = b_0 - b_0 + b_1(\log H_1 - \log H_0)$$
$$+ b_2\left(\frac{100}{A_1} - \frac{100}{A_0}\right) + b_3(\log B_1 - \log B_0)$$

or $$\log\left(\frac{N_1}{N_0}\right) = b_1 \log\left(\frac{H_1}{H_0}\right) + b_2\left(\frac{100}{A_1} - \frac{100}{A_0}\right) + b_3 \log\left(\frac{B_1}{B_0}\right). \tag{19}$$

Thus, the ratio of future to present number of trees can be computed.

Table 7.--Analysis of variance showing the contribution of each variable in the equation,

$$\log N = b_0 + b_1 (\log H) + b_2 \left(\frac{100}{A}\right) + b_3 (\log B).$$

Source of variation	Degrees of freedom	Sum of squares	Mean square
Constant	1	1,745.5131	
Added effect of (log H)	1	32.5501	32.5501*
Added effect of (log B)	1	11.2550	11.2550*
Added effect of (100/A)	1	.7380	.7380*
Residuals	203	7.0718	.0348
Total	207	1,797.1280	

* Significant beyond the 1-percent level.

VI. AVERAGE VOLUME PER TREE

The volume per acre of a stand of timber can be expressed linearly in the logarithmic transformation in terms of $\log D^2$, log H, and log N, as

$$\log V = b_0 + b_1 (\log D^2) + b_2 (\log H) + b_3 (\log N).$$

Computing the coefficients on the basis of data from 207 plots gives

$$\log V = -2.6950 + 1.0203 (\log D^2) + 0.9704 (\log H) + 0.9996 (\log N). \quad (20)$$

It would be expected that the volume per acre of a stand would vary directly with number of trees, for constant average diameters and heights, in which case the coefficient of log N should be 1. The difference between 0.9996 and one is 0.0004, which is not significant when compared with 0.02248, the standard error of 0.9996. Consequently, the coefficient of log N can be taken as 1 and then,

$$\log V - \log N = \log \left(\frac{V}{N}\right) = \log v$$

in which V is volume of the stand and v is the average volume of the N trees in the stand. Hence,

$$\log v = -2.6950 + 1.0203 (\log D^2) + 0.9704 (\log H). \quad (21)$$

The analysis of variance for equation (20) is shown in table 8.

Table 8.--Analysis of variance showing the contribution of each variable in the equation,

$$\log V = b_0 + b_1 (\log D^2) + b_2 (\log H) + b_3 (\log N).$$

Source of variation	Degrees of freedom	Sum of squares	Mean square
Constant	1	2,276.6595	
Added effect of (log H)	1	18.4229	18.4229*
Added effect of (log N)	1	3.7540	3.7540*
Added effect of (log D^2)	1	1.5913	1.5913*
Residuals	203	.4154	.00205
Total	207	2,300.8431	

* Significant beyond the 1-percent level.

VII. BOARD-FOOT/CUBIC-FOOT RATIO

The logarithm of the ratio of board-foot volume in merchantable trees (10 inches and larger) to the total cubic-foot volume of a stand is related linearly to the reciprocal of average diameter, such that

$$\log R = b_0 + b_1\left(\frac{1}{D}\right) + b_2\left(\frac{1}{D^2}\right) .$$

Tree form can be introduced by making the coefficients of $\frac{1}{D}$ and $\frac{1}{D^2}$ functions of height:

$$b_1 = a_0 + a_1 H$$

$$b_2 = c_0 + c_1 H .$$

Substituting,

$$\log R = b_0 + (a_0 + a_1 H)\left(\frac{1}{D}\right) + (c_0 + c_1 H)\left(\frac{1}{D^2}\right)$$

or

$$\log R = b_0 + a_0\left(\frac{1}{D}\right) + a_1\left(\frac{H}{D}\right) + c_0\left(\frac{1}{D^2}\right) + c_1\left(\frac{H}{D^2}\right) . \quad (22)$$

Equation (22) was fitted to the data of 121 plots; the only significant variables were $\left(\frac{1}{D^2}\right)$ and $\left(\frac{H}{D^2}\right)$. The equation appears on page 20, and the contributions of the significant variables in the regression analysis are shown in table 9. For convenience, the variables were transformed as follows:

$$\log (R \times 100) = b_0 + b_1\left(\frac{100}{D^2}\right) + b_2\left(\frac{H}{D^2}\right) . \quad (23)$$

Table 9.--Analysis of variance showing the contribution of each variable in the equation

$$\log (R \times 100) = b_0 + b_1\left(\frac{100}{D^2}\right) + b_2\left(\frac{H}{D^2}\right).$$

Source of variation	Degrees of freedom	Sum of squares	Mean square
Constant	1	608.8781	
Added effect of $100/D^2$	1	22.0571	22.0571*
Added effect of H/D^2	1	1.4955	1.4955*
Residuals	118	4.7598	.0403
Total	121	637.1905	

* Significant beyond the 1-percent level.